

	Page
3.0 Analysis	3.1-1
3.1 General Considerations	1
3.1.1 Philosophy of Analysis Procedures	1
3.1.2 Analysis Methods	1
3.2.1 Theory (Vacant)	3.2-1
3.2.2 Member and Frame Factors (Vacant)	1
3.2.3 Partial Fixity	1
Appendix A	
3.0-A1 Concentrated Load Coefficients — General	
3.0-A2 Concentrated Load Coefficients — Case I	
3.0-A3 Fixed End Moment Coefficient Chart	
3.0-A4 Influence Lines — Two Equal Spans	
3.0-A5 Coefficients and Factors for Double Tapered Members	
3.0-A6 Stiffness Factors for Tapered Members	
3.0-A7 Carry Over Factors for Tapered Members	
3.0-A8 Fixed End Moments for Tapered Members	

3.0 Analysis

3.1 General Considerations

3.1.1 Philosophy of Analysis Procedures

For the design of concrete bridges, in distribution of moments, generally use the gross moment of inertia of the concrete superstructure. In lieu of including the transformed area of steel for columns or other compression members, 120 percent of the gross moment of inertial of columns and other compression members may generally be used.

3.1.2 Analysis Methods

The maximum live load deflection computed shall be in accordance with AASHTO except that the maximum live load deflection in a span shall not exceed 1/1000 and for a cantilever 1/375, regardless of whether the bridge is used by pedestrians.

3-1:V:BDM3

3.2.1 Theory (Vacant)

3.2.2 Member and Frame Factors (Vacant)

3.2.3 Partial Fixity

In general, assume 50 percent fixity of footings except footings on rock shall be 100 percent fixed. For frame analysis, the point of fixity shall normally be taken to be at the approximate center line of footing. For column design, Volume 2 Sheets 9-220 through 9-225 shall be consulted. This shall hold for footings with or without seals. Where superstructures are supported directly on piles, for analyses of the structure the piles may be assumed fixed at a point 5 feet to 10 feet in the ground. For flat slab bridges supported on piling, the piles shall be assumed pinned at the tops. For design of structures with large diameter shafts see Section 9.8

For one column piers assume the footing fully fixed in the direction transverse to the roadway. For loads on one column piers assume the pier acts transversely as a simple cantilever, fixed at the footing, with no allowance for torsional, or lateral stiffness of the superstructure.

3-2:V:BDM3

BRIDGE DESIGN MANUAL

Appendix A

Analysis

Concentrated Load Coefficients — General

Concentrated Load Coefficients for Single Spans With Constant I

k	V _A	Case I		M _S	M _F A	M _F B	V _B	
		M _A	M _B					
.01	.99	.0098	.0001	.0099	.0099	.0050	.01	.99
.02	.98	.0192	.0004	.0196	.0194	.0100	.02	.98
.03	.97	.0282	.0009	.0291	.0287	.0150	.03	.97
.04	.96	.0369	.0015	.0384	.0376	.0200	.04	.96
.05	.95	.0451	.0024	.0475	.0463	.0249	.05	.95
.06	.94	.0530	.0034	.0564	.0547	.0299	.06	.94
.07	.93	.0605	.0046	.0651	.0628	.0348	.07	.93
.08	.92	.0677	.0059	.0736	.0707	.0397	.08	.92
.09	.91	.0745	.0074	.0819	.0782	.0446	.09	.91
.10	.90	.0810	.0090	.0900	.0855	.0495	.10	.90
.11	.89	.0871	.0108	.0979	.0925	.0543	.11	.89
.12	.88	.0929	.0127	.1056	.0993	.0591	.12	.88
.13	.87	.0984	.0147	.1131	.1057	.0639	.13	.87
.14	.86	.1035	.0169	.1204	.1120	.0686	.14	.86
.15	.85	.1084	.0191	.1275	.1179	.0733	.15	.85
.16	.84	.1129	.0215	.1344	.1236	.0780	.16	.84
.17	.83	.1171	.0240	.1411	.1291	.0825	.17	.83
.18	.82	.1210	.0266	.1476	.1343	.0871	.18	.82
.19	.81	.1247	.0292	.1539	.1393	.0916	.19	.81
.20	.80	.1280	.0320	.1600	.1440	.0960	.20	.80
.21	.79	.1311	.0348	.1659	.1485	.1004	.21	.79
.22	.78	.1338	.0378	.1716	.1527	.1047	.22	.78
.23	.77	.1364	.0407	.1711	.1567	.1089	.23	.77
.24	.76	.1386	.0438	.1824	.1605	.1131	.24	.76
.25	.75	.1406	.0469	.1875	.1641	.1172	.25	.75
.26	.74	.1424	.0500	.1924	.1674	.1212	.26	.74
.27	.73	.1439	.0532	.1971	.1705	.1252	.27	.73
.28	.72	.1452	.0564	.2016	.1734	.1290	.28	.72
.29	.71	.1462	.0597	.2059	.1760	.1328	.29	.71
.30	.70	.1470	.0630	.2100	.1785	.1365	.30	.70
.31	.69	.1476	.0663	.2139	.1807	.1401	.31	.69
.32	.68	.1480	.0696	.2176	.1828	.1436	.32	.68
.33	.67	.1481	.0730	.2211	.1846	.1470	.33	.67
.34	.66	.1481	.0763	.2244	.1863	.1503	.34	.66
.35	.65	.1479	.0796	.2275	.1877	.1536	.35	.65
.36	.64	.1475	.0829	.2304	.1889	.1567	.36	.64
.37	.63	.1469	.0862	.2331	.1900	.1597	.37	.63
.38	.62	.1461	.0895	.2356	.1908	.1626	.38	.62
.39	.61	.1451	.0928	.2379	.1915	.1653	.39	.61
.40	.60	.1440	.0960	.2400	.1920	.1680	.40	.60
.41	.59	.1427	.0992	.2419	.1923	.1705	.41	.59
.42	.58	.1413	.1023	.2436	.1924	.1730	.42	.58
.43	.57	.1397	.1054	.2451	.1924	.1752	.43	.57
.44	.56	.1380	.1084	.2464	.1922	.1774	.44	.56
.45	.55	.1361	.1114	.2475	.1918	.1794	.45	.55
.46	.54	.1341	.1143	.2484	.1913	.1813	.46	.54
.47	.53	.1320	.1171	.2491	.1906	.1831	.47	.53
.48	.52	.1298	.1198	.2496	.1897	.1847	.48	.52
.49	.51	.1274	.1225	.2499	.1887	.1862	.49	.51
.50	.50	.1250	.1250	.2500	.1875	.1875	.50	.50
	V _B	M _B	M _A	M _S	M _F B	M _F A	V _A	k
				Case I		Case III	Case II	

Moment = Coeff. x PL

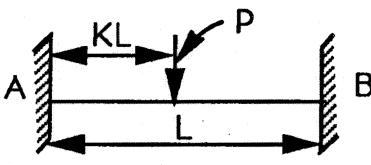
Shear = Coeff. x P

M_S = Simple Beam Moment

= M_A + M_B

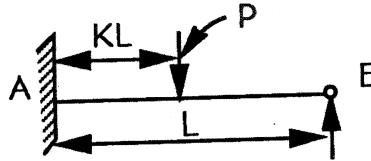
V_A and V_B = Shear at A and B
respectively for simple beam

Case I (Both ends fixed)



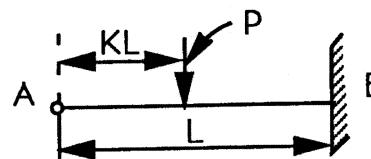
M_A and M_B = Fixed end moment at A and B, respectively

Case II (A end fixed, B end free)



$$\begin{aligned} \underline{M_{F,A}} &= \text{F.E.M. at A} \\ &= M_A + \frac{M_B}{2} = \frac{M_A + M_S}{2} \end{aligned}$$

Case III (A end free, B end fixed)



$$\begin{aligned} \underline{M_{F,B}} &= \text{F.E.M. at B} \\ &= M_B + \frac{M_A}{2} = \frac{M_B + M_S}{2} \end{aligned}$$

BRIDGE DESIGN MANUAL

Appendix A

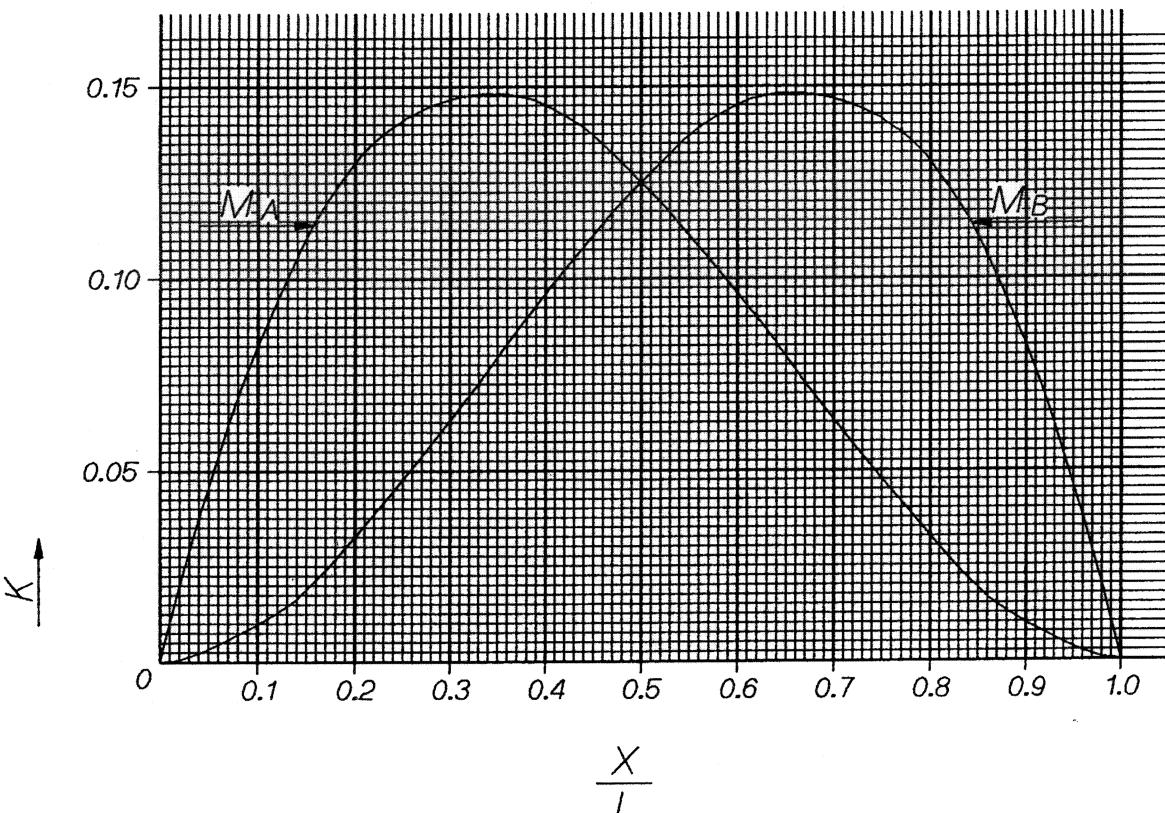
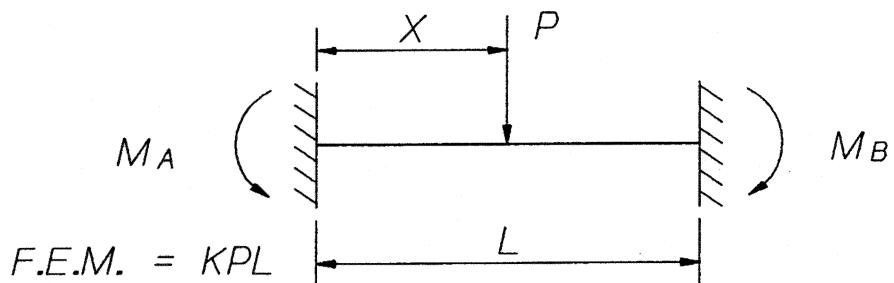
Analysis

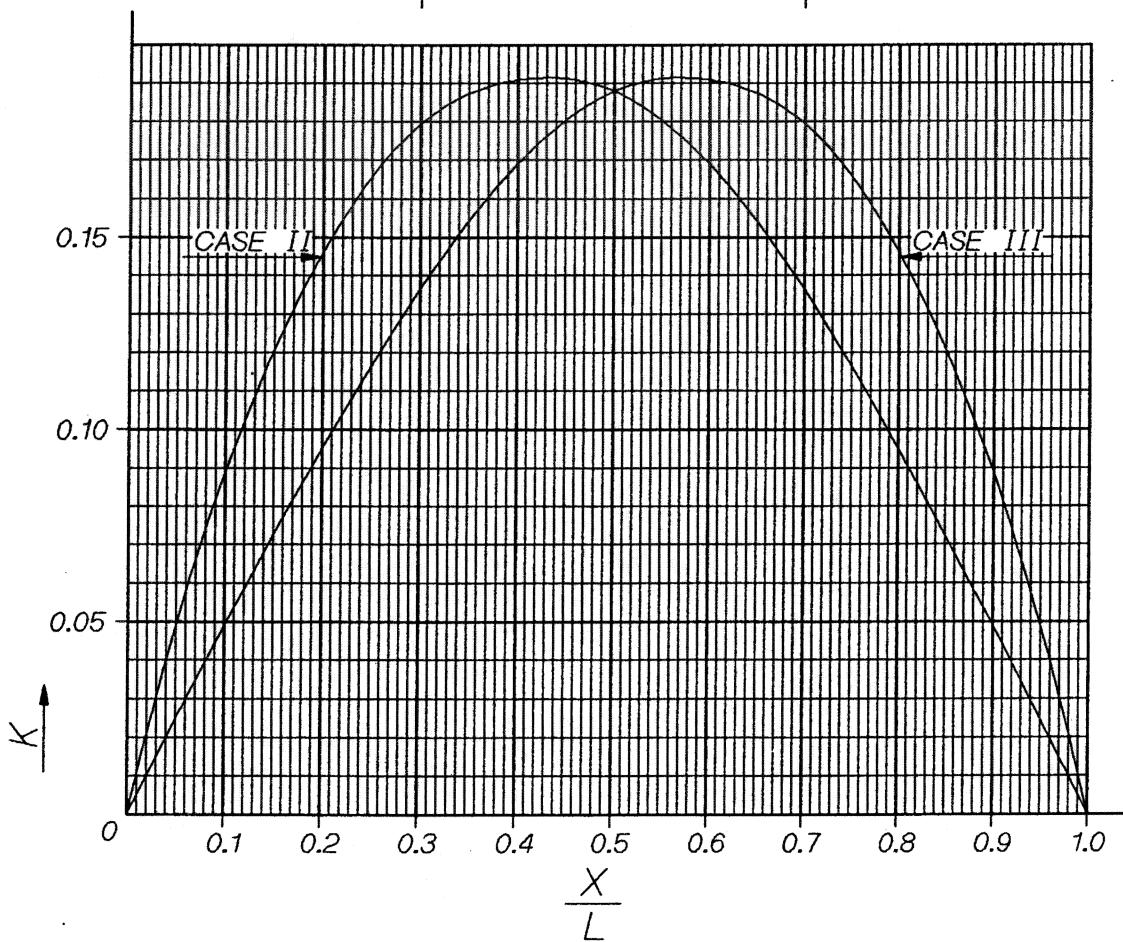
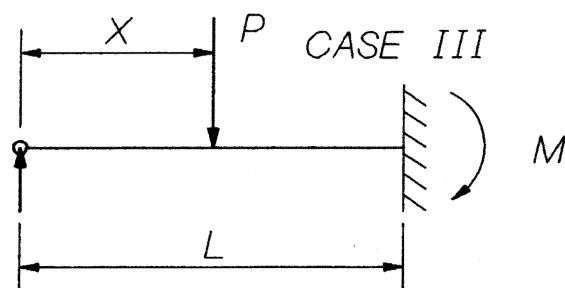
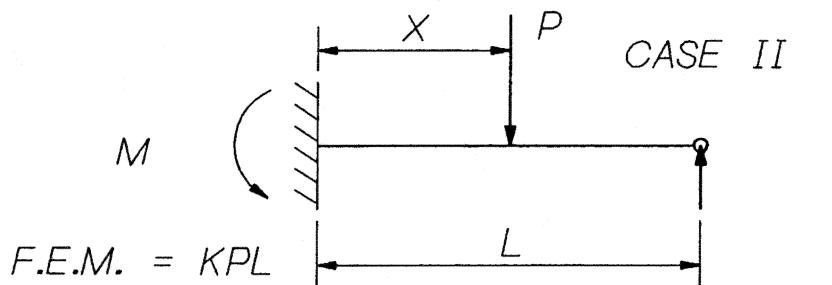
Concentrated Load Coefficients — Case I

Moment = Coeff. x PL												
A			K _A L		K _B L		B					
K _A	K _B	M _A	M _B	K _A	K _B	M _A	M _B	K _A	K _B	M _A	M _B	
.000	1.000	0.0000	0.0000	.100	.900	.0810	.0090	.200	.800	.1280	.0320	
.001	.999	.0010	.0000	.101	.899	.0816	.0092	.201	.795	.1282	.0323	
.002	.998	.0020	.0000	.102	.898	.0823	.0093	.202	.798	.1284	.0326	
.003	.997	.0030	.0000	.103	.897	.0829	.0095	.203	.797	.1285	.0328	
.004	.996	.0040	.0000	.104	.896	.0835	.0097	.204	.796	.1293	.0331	
.005	.995	.0050	.0000	.105	.895	.0841	.0099	.205	.795	.1296	.0334	
.006	.994	.0059	.0000	.106	.894	.0847	.0100	.206	.794	.1299	.0337	
.007	.993	.0069	.0000	.107	.893	.0853	.0102	.207	.793	.1302	.0340	
.008	.992	.0079	.0001	.108	.892	.0859	.0104	.208	.792	.1305	.0343	
.009	.991	.0088	.0001	.109	.891	.0865	.0106	.209	.791	.1308	.0346	
.010	.990	.0098	.0001	.110	.890	.0871	.0108	.210	.790	.1311	.0348	
.011	.989	.0108	.0001	.111	.889	.0877	.0110	.211	.789	.1314	.0351	
.012	.988	.0117	.0001	.112	.888	.0883	.0111	.212	.788	.1316	.0354	
.013	.987	.0127	.0002	.113	.887	.0889	.0113	.213	.787	.1319	.0357	
.014	.986	.0136	.0002	.114	.886	.0895	.0115	.214	.786	.1322	.0360	
.015	.985	.0146	.0002	.115	.885	.0901	.0117	.215	.785	.1325	.0363	
.016	.984	.0155	.0003	.116	.884	.0906	.0119	.216	.784	.1328	.0366	
.017	.983	.0164	.0003	.117	.883	.0912	.0121	.217	.783	.1330	.0369	
.018	.982	.0174	.0003	.118	.882	.0918	.0123	.218	.782	.1333	.0372	
.019	.981	.0183	.0004	.119	.881	.0924	.0125	.219	.781	.1336	.0375	
.020	.980	.0192	.0004	.120	.880	.0929	.0127	.220	.780	.1338	.0378	
.021	.979	.0201	.0004	.121	.879	.0935	.0129	.221	.779	.1341	.0380	
.022	.978	.0210	.0005	.122	.878	.0940	.0131	.222	.778	.1344	.0383	
.023	.977	.0220	.0005	.123	.877	.0946	.0133	.223	.777	.1346	.0386	
.024	.976	.0229	.0006	.124	.876	.0952	.0135	.224	.776	.1349	.0389	
.025	.975	.0238	.0006	.125	.875	.0957	.0137	.225	.775	.1351	.0392	
.026	.974	.0247	.0007	.126	.874	.0962	.0139	.226	.774	.1354	.0395	
.027	.973	.0256	.0007	.127	.873	.0968	.0141	.227	.773	.1356	.0398	
.028	.972	.0265	.0008	.128	.872	.0973	.0143	.228	.772	.1358	.0401	
.029	.971	.0273	.0008	.129	.871	.0979	.0145	.229	.771	.1361	.0404	
.030	.970	.0282	.0009	.130	.870	.0984	.0147	.230	.770	.1364	.0407	
.031	.969	.0291	.0009	.131	.869	.0989	.0149	.231	.769	.1366	.0410	
.032	.968	.0300	.0010	.132	.868	.0995	.0151	.232	.768	.1368	.0413	
.033	.967	.0309	.0011	.133	.867	.1000	.0153	.233	.767	.1371	.0416	
.034	.966	.0317	.0011	.134	.866	.1005	.0155	.234	.766	.1373	.0419	
.035	.965	.0326	.0012	.135	.865	.1010	.0158	.235	.765	.1375	.0422	
.036	.964	.0335	.0012	.136	.864	.1015	.0160	.236	.764	.1378	.0426	
.037	.963	.0343	.0013	.137	.863	.1020	.0162	.237	.763	.1380	.0429	
.038	.962	.0352	.0014	.138	.862	.1025	.0164	.238	.762	.1382	.0432	
.039	.961	.0360	.0015	.139	.861	.1030	.0166	.239	.761	.1384	.0435	
.040	.960	.0369	.0015	.140	.860	.1035	.0169	.240	.760	.1386	.0438	
.041	.959	.0377	.0016	.141	.859	.1040	.0171	.241	.759	.1388	.0441	
.042	.958	.0385	.0017	.142	.858	.1045	.0173	.242	.758	.1390	.0444	
.043	.957	.0394	.0018	.143	.857	.1050	.0175	.243	.757	.1393	.0447	
.044	.956	.0402	.0019	.144	.856	.1055	.0178	.244	.756	.1395	.0450	
.045	.955	.0410	.0019	.145	.855	.1060	.0180	.245	.755	.1397	.0453	
.046	.954	.0419	.0020	.146	.854	.1065	.0182	.246	.754	.1399	.0456	
.047	.953	.0427	.0021	.147	.853	.1070	.0184	.247	.753	.1401	.0459	
.048	.952	.0435	.0022	.148	.852	.1070	.0187	.248	.752	.1402	.0463	
.049	.951	.0443	.0023	.149	.851	.1079	.0189	.249	.751	.1404	.0466	
.050	.950	.0451	.0024	.150	.850	.1081	.0191	.250	.750	.1406	.0469	
.051	.949	.0459	.0025	.151	.849	.1088	.0194	.251	.749	.1408	.0472	
.052	.948	.0467	.0026	.152	.848	.1093	.0196	.252	.748	.1410	.0475	
.053	.947	.0475	.0027	.153	.847	.1098	.0198	.253	.747	.1412	.0478	
.054	.946	.0483	.0028	.154	.846	.1102	.0201	.254	.746	.1414	.0481	
.055	.945	.0491	.0029	.155	.845	.1107	.0203	.255	.745	.1415	.0484	
.056	.944	.0499	.0030	.156	.844	.1111	.0205	.256	.744	.1417	.0488	
.057	.943	.0507	.0031	.157	.843	.1116	.0208	.257	.743	.1419	.0491	
.058	.942	.0515	.0032	.158	.842	.1120	.0210	.258	.742	.1420	.0494	
.059	.941	.0522	.0033	.159	.841	.1125	.0213	.259	.741	.1422	.0497	
.060	.940	.0530	.0034	.160	.840	.1129	.0215	.260	.740	.1424	.0500	
.061	.939	.0538	.0035	.161	.839	.1133	.0217	.261	.739	.1425	.0503	
.062	.938	.0546	.0036	.162	.838	.1138	.0220	.262	.738	.1427	.0507	
.063	.937	.0554	.0037	.163	.837	.1142	.0222	.263	.737	.1429	.0510	
.064	.936	.0561	.0038	.164	.836	.1146	.0225	.264	.736	.1430	.0513	
.065	.935	.0568	.0040	.165	.835	.1150	.0227	.265	.735	.1432	.0516	
.066	.934	.0576	.0041	.166	.834	.1155	.0230	.266	.734	.1433	.0519	
.067	.933	.0583	.0042	.167	.833	.1159	.0232	.267	.733	.1435	.0523	
.068	.932	.0591	.0043	.168	.832	.1163	.0235	.268	.732	.1436	.0526	
.069	.931	.0598	.0044	.169	.831	.1168	.0237	.269	.731	.1437	.0529	
.070	.930	.0605	.0046	.170	.830	.1171	.0240	.270	.730	.1439	.0532	
.071	.929	.0613	.0047	.171	.829	.1175	.0242	.271	.729	.1440	.0535	
.072	.928	.0620	.0048	.172	.828	.1179	.0245	.272	.728	.1442	.0539	
.073	.927	.0627	.0049	.173	.827	.1183	.0248	.273	.727	.1443	.0542	
.074	.926	.0635	.0051	.174	.826	.1187	.0250	.274	.726	.1444	.0545	
.075	.925	.0642	.0052	.175	.825	.1191	.0253	.275	.725	.1445	.0548	
.076	.924	.0649	.0053	.176	.824	.1195	.0255	.276	.724	.1447	.0552	
.077	.923	.0656	.0055	.177	.823	.1199	.0258	.277	.723	.1448	.0555	
.078	.922	.0663	.0056	.178	.822	.1203	.0260	.278	.722	.1449	.0558	
.079	.921	.0670	.0057	.179	.821	.1207	.0263	.279	.721	.1450	.0561	
.080	.920	.0677	.0059	.180	.820	.1210	.0266	.280	.720	.1452	.0564	
.081	.919	.0684	.0060	.181	.819	.1214	.0268	.281	.719	.1453	.0568	
.082	.918	.0691	.0062	.182	.818	.1218	.0271	.282	.718	.1454	.0571	
.083	.917	.0698	.0063	.183	.817	.1222	.0274	.283	.717	.1455	.0574	
.084	.916	.0705	.0065	.184	.816	.1225	.0276	.284	.716	.1456	.0577	
.085	.915	.0712	.0067	.185	.815	.1229	.0279	.285	.715	.1457	.0581	
.086	.914	.0718	.0068	.186	.814	.1232	.0282	.286	.714	.1458	.0584	
.087	.913	.0725	.0069	.187	.813	.1236	.0284	.287	.713	.1459	.0587	
.088	.912	.0732	.0071	.188	.812	.1240	.0287	.288	.712	.1460	.0591	
.089	.911	.0739	.0072	.189	.811	.1243	.0290	.289	.711	.1461	.0594	
.090	.910	.0745	.0074	.190	.810	.1247	.0292	.290	.710	.1462	.0597	
.091	.909	.0752	.0075	.191	.809	.1250	.0295	.291	.709	.1463	.0600	
.092	.908	.0759	.0077	.192	.808	.1253	.0298	.292	.708	.1464	.0604	
.093	.907	.0765	.0078	.193	.807	.1257	.0301	.293	.707	.1465	.0607	
.094	.906	.0772	.0080	.194	.806	.1260	.0303	.294	.706	.1465	.0610	
.095	.905	.0778	.0082	.195	.805	.1264	.0306	.295	.705	.1466	.0614	
.096	.904	.0785	.0083	.196	.804	.1267	.0309	.296	.704	.1467	.0617	
.097	.903	.0791	.0085	.197	.803	.1270	.0312	.297	.703	.146		

F.E.M. COEFFICIENTS

CASE I



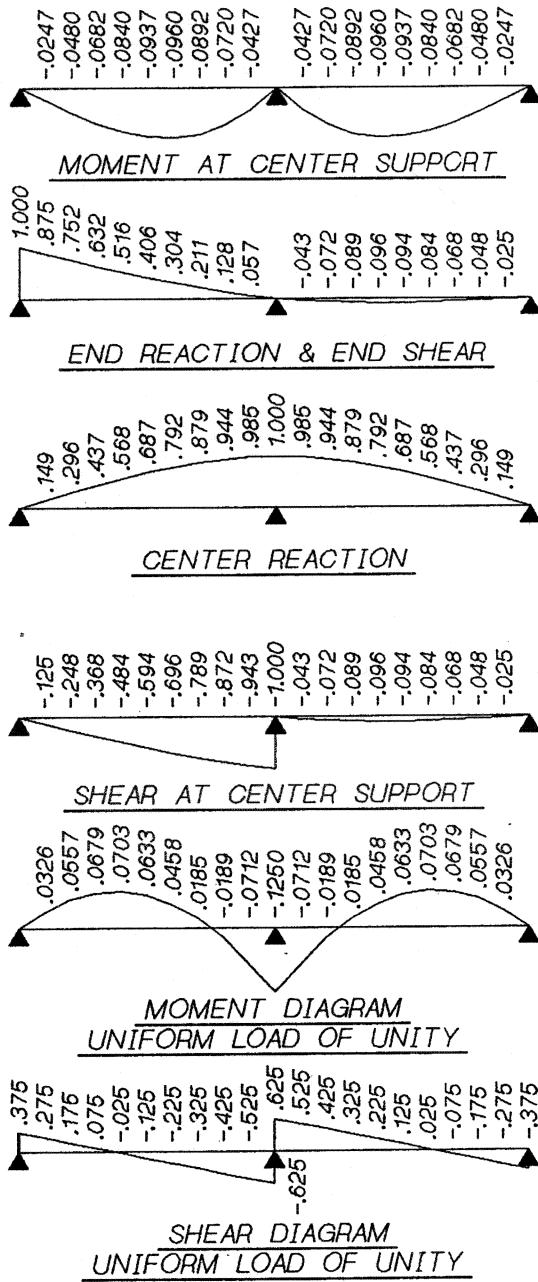
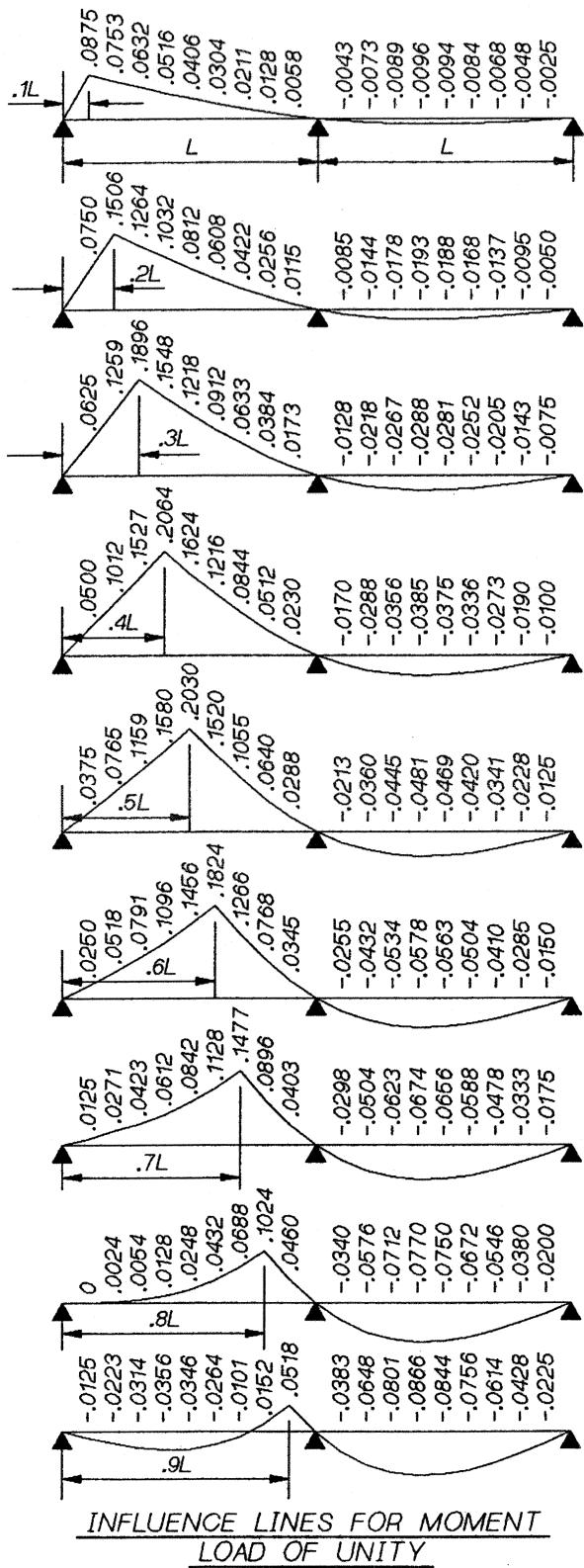
F.E.M. COEFFICIENTS

BRIDGE DESIGN MANUAL

Appendix A

Analysis

Influence Lines — Two Equal Spans



Appendix A

Analysis

Coefficients and Factors
for Double Tapered Members

Tapered structural members of rectangular cross-section are often used in concrete structures, for example, as supporting legs for viaducts on expressways. Charts for determining the moment distribution factors for this type of member are found in the Portland Cement Association's *Handbook of Frame Constants*, and in many textbooks on statically indeterminate structures.

However, these charts have limitations in actual application. The proportions of a structural member may be such that it lies outside the range of these charts. Even if the member lies within the limits of the charts, interpolation may be required and this is not always satisfactory because the stiffness factor of such a member is very sensitive to a small change in dimensions.

Following the principles used in constructing the design charts, general formulas for computing carry-over factors, stiffness factors and fixed-end moments due to support movements are here derived for tapered members of rectangular cross-section. The equations can be programmed for electronic computers.

In Fig. 1, the dimensional nomenclature is given for tapered members whose faces have slopes of $s_1 = rh/(2L)$ and $s_2 = qb/(2L)$. Let the constants C_1, C_2, C_3, C_4 and D be so defined that

$$C_1 = \frac{r^2 + 2q + rq}{2(r-q)^2(1+r)^2}$$

$$C_2 = \frac{q - 3r - 2}{2(r-q)^2(1+r)^2}$$

$$C_3 = \frac{r + 2 + q}{2(r-q)^2(1+r)} = C_1 - C_2$$

$$C_4 = \frac{-r(-r) - 2 - 3q(1+r)}{2(r-q)^2(1+r)}$$

$$D = \frac{\log_e(1+r) - \log_e(1+q)}{(r-q)^2}$$

I. CASE FOR FULLY RESTRAINED MEMBER WHEN $r \neq q$.

1. Carry-over factors. Note that the signs for C_{AB} and C_{BA} should be positive. Their values are:

$$C_{BA} = \frac{(C_1 - C_2) - (q + 1)D}{C_2 + D}$$

$$C_{AB} = \frac{C_3 - (q + 1)D}{C_4 + (q + 1)^2 D}$$

2. Stiffness factors. Let $I_c = bb^3/12$, and express the stiffness, $K = (k EI_c)/L$. For members AB fixed at both ends,

$$K_{AB} = (k_{AB} EI_c)/L, \text{ and}$$

$$K_{BA} = (k_{BA} EI_c)/L$$

for which

$$k_{BA} = \frac{1}{[C_4 + (q + 1)^2 D] - C_{BA} [C_3 - (q + 1)D]}$$

$$k_{AB} = \frac{1}{[C_2 + D] - C_{AB} [C_3 - (q + 1)D]}$$

MOMENT DISTRIBUTION FACTORS
FOR TAPERED MEMBERS

3. Fixed-end moments due to lateral displacement. Let the lateral displacement of the fixed-end member be Δ , and let $R = \Delta/L$. The fixed-end moment due to Δ becomes:

$$M_{AB} = K_{AB} (1 + C_{AB}) R$$

$$M_{BA} = K_{BA} (1 + C_{BA}) R$$

II. CASE FOR FULLY RESTRAINED MEMBER WHEN $r = q$.

1. Carry-over factors

$$C_{BA} = \frac{1+r}{2},$$

$$C_{AB} = \frac{1}{2(1+r)}$$

2. Stiffness factor

$$k_{BA} = 4(1+r), k_{AB} = 4(1+r)^2$$

$$K = k EI_c/L.$$

3. Fixed-end moments due to lateral displacement

$$M_{BA} = 2(1+r)(3+r)R EI_c/L.$$

$$M_{AB} = 2(1+r)^2(3+2r)R EI_c/L.$$

Two numerical examples are given to illustrate the use of the formulas. It is necessary to mention only that the formulas for carry-over factors involve the difference of two large numbers of the same order of magnitude. To obtain three significant figures in the final results, six or seven figures should be used for computing the constants. This explains why extrapolation from design charts is not always satisfactory.

Example 1. For member AB, fixed at both ends, $L = 30$ ft, $h = 3$ ft, $b = 2$ ft, $r = 0.6$, $q = 1$, and $E = 3 \times 10^3$ ksi. Then, $I_c = 4.5$ ft 4 , $s_1 = 1/33.3$, $s_2 = 1/30$.

Then, by substitution in the formulas:

$$C_{BA} = 0.845 \quad C_{AB} = 0.295$$

$$k_{BA} = 6.78 \quad k_{AB} = 19.41$$

$$K_{BA} = 438,000 \text{ kip-ft per radian}$$

$$K_{AB} = 1,280,000 \text{ kip-ft per radian}$$

Example 2. Here again, member AB is fixed at both ends and $L = 30$ ft, $h = 3$ ft, $b = 2$ ft. In this case however, $r = q = 1$. As in Example 1, $E = 3 \times 10^3$ ksi; $I_c = 4.5$ ft 4 ; $s_1 = 1/20$; and $s_2 = 1/30$.

Again by substitution,

$$C_{BA} = 1.0 \quad C_{AB} = 0.25$$

$$k_{BA} = 8.0 \quad k_{AB} = 32.0$$

$$K_{BA} = 520,000 \text{ kip-ft per radian}$$

$$K_{AB} = 2,080,000 \text{ kip-ft per radian}$$

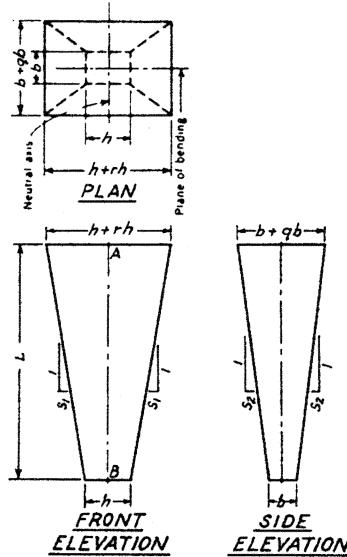


FIG. 1. Dimensional nomenclature for tapered members of rectangular cross-section.

BRIDGE DESIGN MANUAL

Appendix A

Analysis

Coefficients and Factors for Double Tapered Members

